Title: Exploring the Altitude Drawn to the Hypotenuse of a Right Triangle

Brief Overview:

Using Geometer's Sketchpad, students will be able to demonstrate their understanding of the theorem: "If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original and to each other." They will apply previously learned concepts which include translations, reflections, rotations, and vectors. They will also review vocabulary used with a right triangle.

Link to Standards:

- **Problem Solving** Students will demonstrate an understanding of the Theorem by using Geometer's Sketchpad to explore the relationships between the measures of angles and sides of resulting triangles.
- **Reasoning** Students will demonstrate their ability to reason logically through manipulation of the triangles from the different orientations of the original positions to show that they are similar.
- **Number Relationships** Students will demonstrate their ability to use Geometer's Sketchpad to measure and compare lengths of segments. (Application of scale factor)

Grade/Level:

Grades 9 - 12, Geometry

Duration/Length:

This lesson will take one or two 90 minute periods.

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Geometer's Sketchpad
- Transformations
- Vectors
- Similarity
- Scale factor
- Triangle congruence

Objectives:

Students will:

- work cooperatively with a partner.
- discuss the lab and give appropriate support for responses.
- apply previously learned concepts to illustrate conclusions.
- write a description of the relationships among the resulting triangles.

Materials/Resources/Printed Materials:

- Geometer's Sketchpad Software Package
- Teacher's Guide
- Student Lab Sheets
- Follow-up activity
- Extension activity sheet

Development/Procedures:

- Construct a rectangle with a diagonal.
- Construct a perpendicular to the hypotenuse of one right triangle.
- Follow instructions to translate triangles.
- Complete student worksheet.
- Make hard copy of final construction.

Evaluation:

The teacher will circulate about the classroom to observe student progress and to make sure all students are on task. The finished product which has been produced using Geometer's Sketchpad will be printed. The completed lab will be collected. Assessment of understanding will be based on Follow-Up activity.

Extension:

Following the investigation lab, the teacher will discuss the geometric mean and ask students to complete the activities listed below.

Activity 1: Use Geometer's Sketchpad to show that the length of the altitude to the hypotenuse of a right triangle cannot be the geometric mean between the lengths of the legs of the triangle.

Activity 2: Use Geometer's Sketchpad to show that the length of the altitude from the right angle to the hypotenuse is the geometric mean of the lengths of the two segments of the hypotenuse.

Activity 3: Use Geometer's Sketchpad to show that if the altitude is drawn to the hypotenuse of a right triangle, then the length of either leg is the geometric mean of the length of the hypotenuse and the length of the segment of the hypotenuse adjacent to that leg.

Reference:

This activity was developed directly from a lesson investigation on page 428 in the book Geometry: An Integrated Approach by Larson, Boswell, and Stiff.

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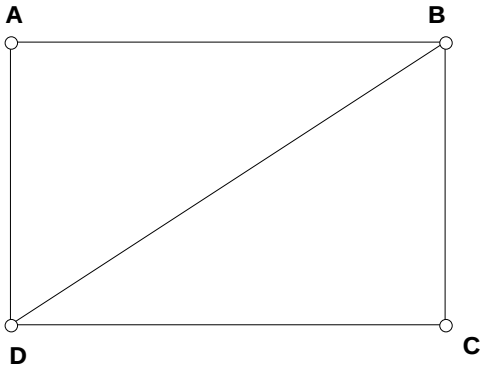
Name_____
Name_____
Date_____Period_____

PARTNER EXPLORATION
INVESTIGATING AN ALTITUDE DRAWN TO A HYPOTENUSE

Part I. Constructing the Figure

A. Construct a rectangle with one diagonal

1. Use tool to make a line segment
2. Use ***construct perpendicular line*** to complete the rectangle (Note: ***Construct perpendicular line*** will ensure that the angles of the rectangle are always 90° , even if it is dragged by a corner.)
3. Select lines, go to ***display hide lines*** to hide unnecessary lines then select points and go to construct ***segment*** to connect the points of the rectangle.
4. Using hand tool, label as shown below.



Investigate:

Are the two right triangles formed congruent? Illustrate using drawing above and explain.

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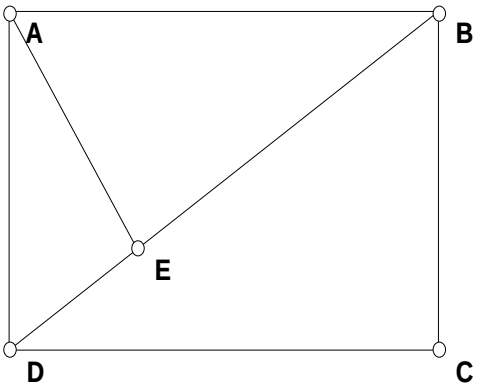
B. Construct an altitude from A to DB

1. Select A and DB and use ***construct perpendicular line*** to make altitude. Hold shift and select DB then go to ***construct point at intersection.***

2. Use ***display hide lines*** then select points A and E, go to ***construct segment*** to obtain diagram as shown below. Label as shown.



Investigate:

What is the relationship between the area of triangle BCD and the sum of the areas of triangles ADE and ABE ?

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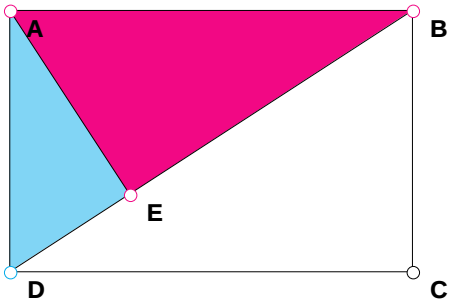
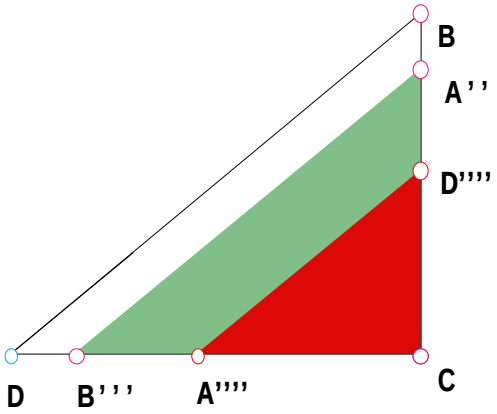
Classify the triangles by their angles. What types of triangles are present in this figure?_____
_____

3. Select the 3 points of triangle ADE (hold shift)
4. Use ***display color,*** pick color of choice
5. Use ***construct polygon interior*** to shade the triangle
6. Do the same thing to triangle ABE in a different color to obtain the figure.



Part II. Manipulating the Figure

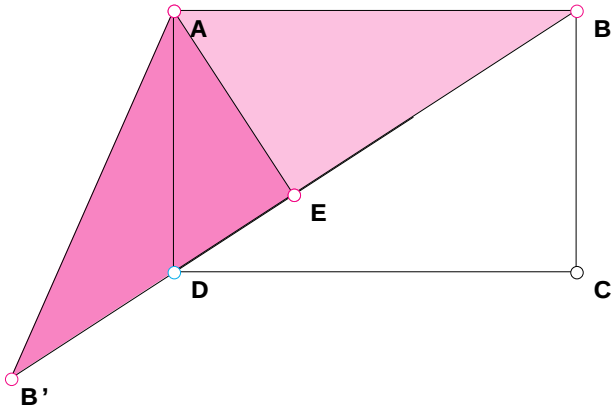
In this part of the lab you will manipulate the figure above so that you will create the figure below as a final product.



A. Manipulating triangle ABE

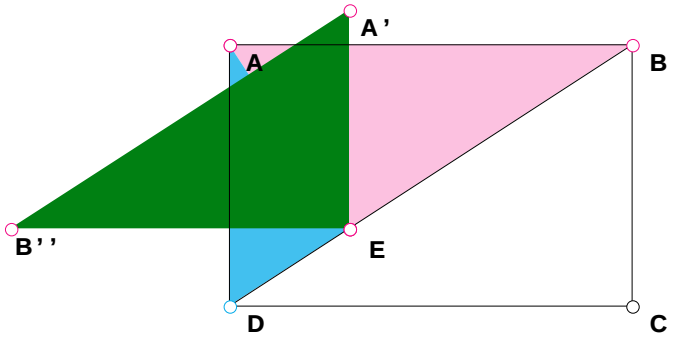
Procedure:

1. Select AE and go to ***transform mark mirror.***
2. Select points A, B, E, and triangle ABE.
3. Use ***transform reflect***. Your figure should look like the next drawing. Label as indicated.



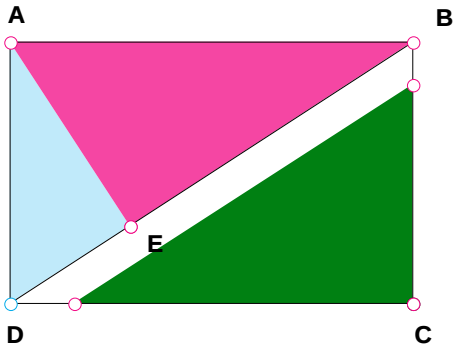
4. Select points B, D, C, in that order. Go to ***transform mark angle***. Select point E and go to ***transform mark center***.

5. Select the points A, E, B' and the triangle. Use ***transform rotate*** (***by marked angle***) to get the next figure. Go to ***display hide polygon.***



6. Select points E and C (in that order). Go to ***transform marked vector*** (from E to C).

7. Select points A', B'' and triangle A'B''E and go to ***transform translate*** (***by marked vector***). Select points A' and B'' and triangle A'B''E. Go to ***display hide objects*** to obtain the figure below.



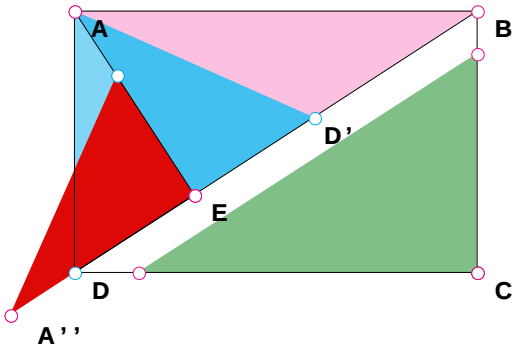
B. Manipulating triangle AED

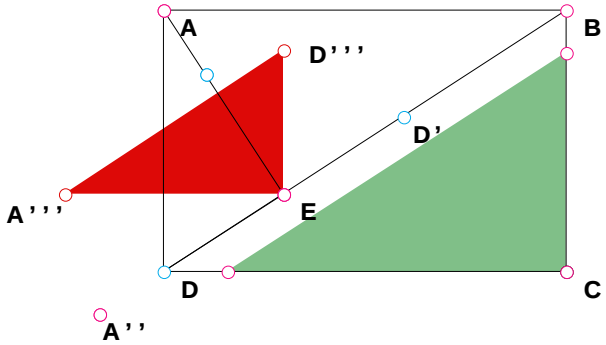
Procedure:

1. Select points A, E, D and click on triangle AED. Use ***transform reflect*** (to reflect triangle AED across already marked mirror AE.

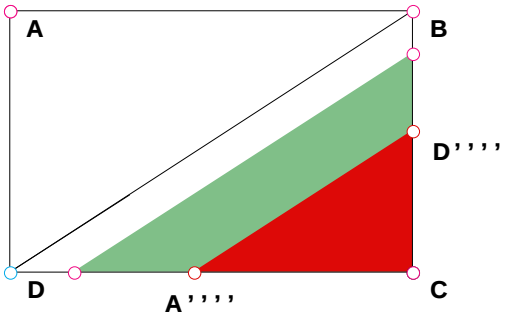
2. Use ***transform rotate*** (***by fixed angle***). Type 90°. Triangle AED will rotate about already marked center E to obtain the next figure on the left. Then hide polygon. Label.

3. Select points A" and D" and triangle A"D"E. Go to ***transform rotate (by marked angle).*** Next, ***display hide polygon*** A"D"E, to obtain figure below on the right.

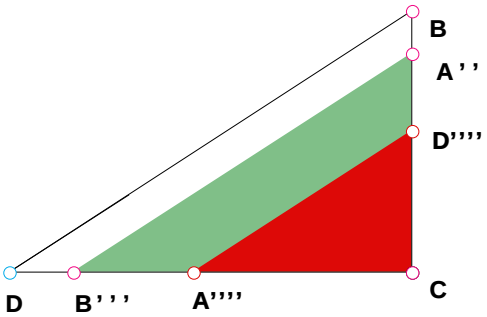




4. Select points A"' and D"' and triangle A"'D"'E. Use ***transform translate*** (***by marked vector***) . Hide triangle A"'D"'E to obtain the figure below.



5. Select AB and AD. Go to ***display hide.*** Label as indicated. The final figure is three overlapping triangles as shown below.



Investigation:

Compare the resulting three triangles and make a prediction about the relationship of the corresponding angles.

Procedure:

You will now measure each angle of the three triangles to determine if your prediction is correct. To do this, select three points with the vertex of an angle in the center. Use measure angle . Complete the chart below.

Measure of the Three Angles of the Triangle

Triangle BCD (large)			
Triangle B'''CA''(med)			
Triangle A''''CD''''(small)			

Does the information in the chart confirm your prediction? Explain.

Make a prediction about the relationship of the sides of the three triangles.

Procedure:

Select the endpoints of the side of a triangle. Go to construct segment followed by measure length. Complete the chart below.

Measures of the Sides of the Triangles

	Side 1	Side 2	Side 3
Triangle BCD			
Triangle B'''CA''			
Triangle A''''CD''''			

Does the information in the chart confirm your prediction? Explain. (Hint: Use scale factor.)

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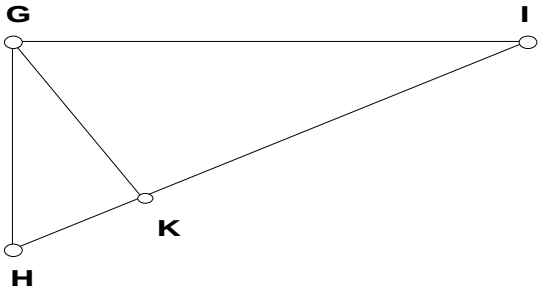
FOLLOWING UP ON THE INVESTIGATION



In the diagram above triangle HGI is a right triangle. GK is an altitude drawn to the hypotenuse. Discuss all relationships among the three resulting triangles.

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